2103000206020035
EXAMINATION FEBRUARY-MARCH 2024
BACHELOR OF SCIENCE (SIXTH SEMESTER)
MATHEMATICS-X (MTH-605-DISCRETE MATHEMATICS)LEVEL 2

[Time: As Per Schedule] **[Max. Marks: 50]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination : **BACHELOR OF SCIENCE (SIXTH SEMESTER)**
   b. Name of the Subject : **MATHEMATICS-X (MTH-605-DISCRETE MATHEMATICS) LEVEL 2**
   c. Subject Code No : **2103000206020035**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations and conversations.

Seat No:

Student's Signature

Q.1 Answer the following (any five) : **10**

1. Draw the circuit diagram of the Boolean function $f(a, b, c) = abc + ab\bar{c} + a\bar{b}c$

2. Define: Sub lattice and Distributive Lattice.

3. Show that in a complemented distributive lattice $a' \oplus b = 1 \Leftrightarrow b' \leq a'$

4. If $X = \{1,2,3,4\}$ and $R\{\langle 1,1\rangle, \langle 1,4\rangle, \langle 4,1\rangle, \langle 4,4\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,3\rangle\}$, then write the matrix of R and sketch its graph.

5. Obtain sum of products canonical form of $x_1 * x_2$ into three variables.

6. Obtain the values of the Boolean forms: $x_1' * (x_1 \oplus x_2)$ over the ordered pairs of the two-elemented Boolean algebra.

7. Minimized the Boolean expression $\sum(0,1,5)$ with three variable K-map.

8. Draw the Hasse diagram of $\{3,9,27,54\}$ under the partial ordering relation "divides". Is this set totally ordered? Justify your answer.

Q.2 Answer the following: (Any Two) **10**

1. If $X = \{1,2,3,4\}$ and $R\{\langle 1,1\rangle, \langle 1,4\rangle, \langle 4,1\rangle, \langle 4,4\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,3\rangle\}$, then write the matrix of R and sketch its graph. Is R transitive relation? Justify your answer.

2. If R is a partial ordering relation on a set X and $A \subseteq X$, show that $R \cap (A \times A)$ is a partial ordering relation on a set A.

3. Show that the relation "congruence modulo m" given by $\equiv = \{\langle x, y\rangle / x - y \text{ is divisible by } m\}$ over the set of positive integers is an equivalence relation. Also show that if $x_1 \equiv y_1$ and $x_2 \equiv y_2$ then $(x_1 + x_2) \equiv (y_1 + y_2)$.

Q.3 Answer the following: (Any Two) **10**

1. Show that in a Lattice with two or more elements, no element is its own complement.

2. Let $L = \{0,1\}$ then draw the Hasse diagram of $\langle L^3, \leq_3\rangle$, and show that $\langle L^3, \leq_3\rangle$ is a complemented lattice.

3. Show that in a distributive lattice, the distributive laws can be generalized as

$$a * \left(\bigoplus_{i=1}^{n} bi \right) = \bigoplus_{i=1}^{n} (a * b_i) \;\&\; a \oplus \left(\underset{i=1}{\overset{n}{*}} bi \right) = \underset{i=1}{\overset{n}{*}} (a \oplus b_i)$$

Q.4 Answer the following: (Any Two) **10**

1. Check whether the following Boolean expressions are equivalent to one another or not.
   (i) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$
   (ii) $(x \oplus y) * (x' \oplus z)$

2. Obtain product of sums canonical form for the expression $x_1 x_2' + x_3$

3. Define sub-Boolean algebra. If $\langle B, *, \oplus, ', 0,1\rangle$ is a Boolean algebra then show that $S \subseteq B$ is sub-Boolean algebra if S is closed with respect to the set of operations $\{\oplus, '\}$.

Q.5 Answer the following: (Any Two) **10**

1. Use Karnaugh map representation to find minimal expression of the Boolean function $f(a, b, c) = \sum(0,1,2,4,5,6)$.

2. Use the Quine-Mc-Cluskey method to obtain minimal expression for the Boolean function $f(a, b, c, d) = \sum(0,1,3,7,8,9,11,15)$

3. Find the Prime Implicants and Essential prime Implicants of the Boolean expression $f(A, B, C) = \sum(0,1,3,4,5,6,7)$

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